LEMMA

Suppose G is an (r, Δ, c) - boundary expander and $T \subseteq V_{exp}(G)$ has size $|T| \le k \le r$; Then $|\text{closure}_{r,\nu}(T)| < \dfrac{k\Delta}{c-\nu}$.

Proof :

Let $C(T) = \text{Closure}_{r,\nu}(T)$

From def of closure: $T \subseteq C(T)$ with $C(T) \lesssim r$

$$|\partial(C(T)) \setminus N(T)| < \nu \cdot |C(T)|$$

But by boundary expansion for any $S \subseteq V_{exp}(G)$ with $|S| \le r$

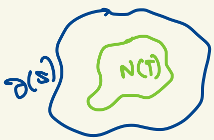
$$|\partial(S)| \ge c \cdot |S|$$

Setting $S = C(T)$

we get $|\partial(C(T))| \ge c \cdot |C(T)|$

counting → most N(T) can take from ∂(S)

$$|\partial(S) \setminus N(T)| \ge |\partial(S)| - |N(T)|$$
$$\ge \underbrace{c \cdot |S|}_{\text{by boundary expansion}} - \underbrace{k \cdot \Delta}_{\text{max num neighbours for } T}$$



$\therefore \quad c \cdot |S| - k\Delta < \nu \cdot |S|.$

$\downarrow$

$|\text{closure}_{r,\nu}(T)|$ solve for $|S|$ to get what you need.